

IDENTIDADES TRIGONOMÉTRICAS DE ÁNGULOS COMPUESTOS

Para la suma:

$$\text{Sen}(x+y) = \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x$$

$$\text{Cos}(x+y) = \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

Para la diferencia

$$\text{Sen}(x-y) = \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x$$

$$\text{Cos}(x-y) = \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Observaciones

Si: $a\text{Sen}x + b\text{Cos}x = c$

Donde: $a^2 + b^2 = c^2 \rightarrow \text{Sen}x = \frac{a}{c} \quad \text{y} \quad \text{Cos}x = \frac{b}{c}$

IDENTIDADES TRIGONOMÉTRICAS AUXILIARES DE ÁNGULOS COMPUESTOS

$$\text{Sen}(x+y) \cdot \text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$$

$$\text{Cos}(x+y) \cdot \text{Cos}(x-y) = \text{Cos}^2x - \text{Sen}^2y$$

$$\text{Tan}x + \text{Tan}y + \text{Tan}(x+y) \cdot \text{Tan}x \cdot \text{Tan}y = \text{Tan}(x+y)$$

$$\text{Tan}x - \text{Tan}y - \text{Tan}(x-y) \cdot \text{Tan}x \cdot \text{Tan}y = \text{Tan}(x-y)$$

$$\text{Tan}x + \text{Tan}y = \text{Sen}(x+y) / \text{Cos}x \cdot \text{Cos}y$$

$$\text{Tan}x - \text{Tan}y = \text{Sen}(x - y) / \text{Cos}x \cdot \text{Cos}y$$

$$\text{Ctg}x + \text{Ctg}y = \text{Sen}(x+y) / \text{Sen}x \cdot \text{Sen}y$$

$$\text{Ctg}x - \text{Ctg}y = \text{Sen}(x - y) / \text{Sen}x \cdot \text{Sen}y$$

$$\text{Sen}(x+y+z) = \text{Sen}x \text{Cos}y \text{Cos}z + \text{Sen}y \text{Cos}x \text{Cos}z + \text{Sen}z \text{Cos}x \text{Cos}y - \text{Sen}x \text{Sen}y \text{Sen}z$$

$$\text{Cos}(x+y+z) = \text{Cos}x \text{Cos}y \text{Cos}z - \text{Cos}x \text{Sen}y \text{Sen}z - \text{Cos}y \text{Sen}x \text{Sen}z - \text{Cos}z \text{Sen}x \text{Sen}y$$

$$\text{Tan}(x+y+z) = \frac{\text{Tan}x + \text{Tan}y + \text{Tan}z - \text{Tan}x \text{Tan}y \text{Tan}z}{1 - \text{Tan}x \text{Tan}y - \text{Tan}y \text{Tan}z - \text{Tan}x \text{Tan}z}$$

$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Calcule el valor de: $\text{Sen}16^\circ$

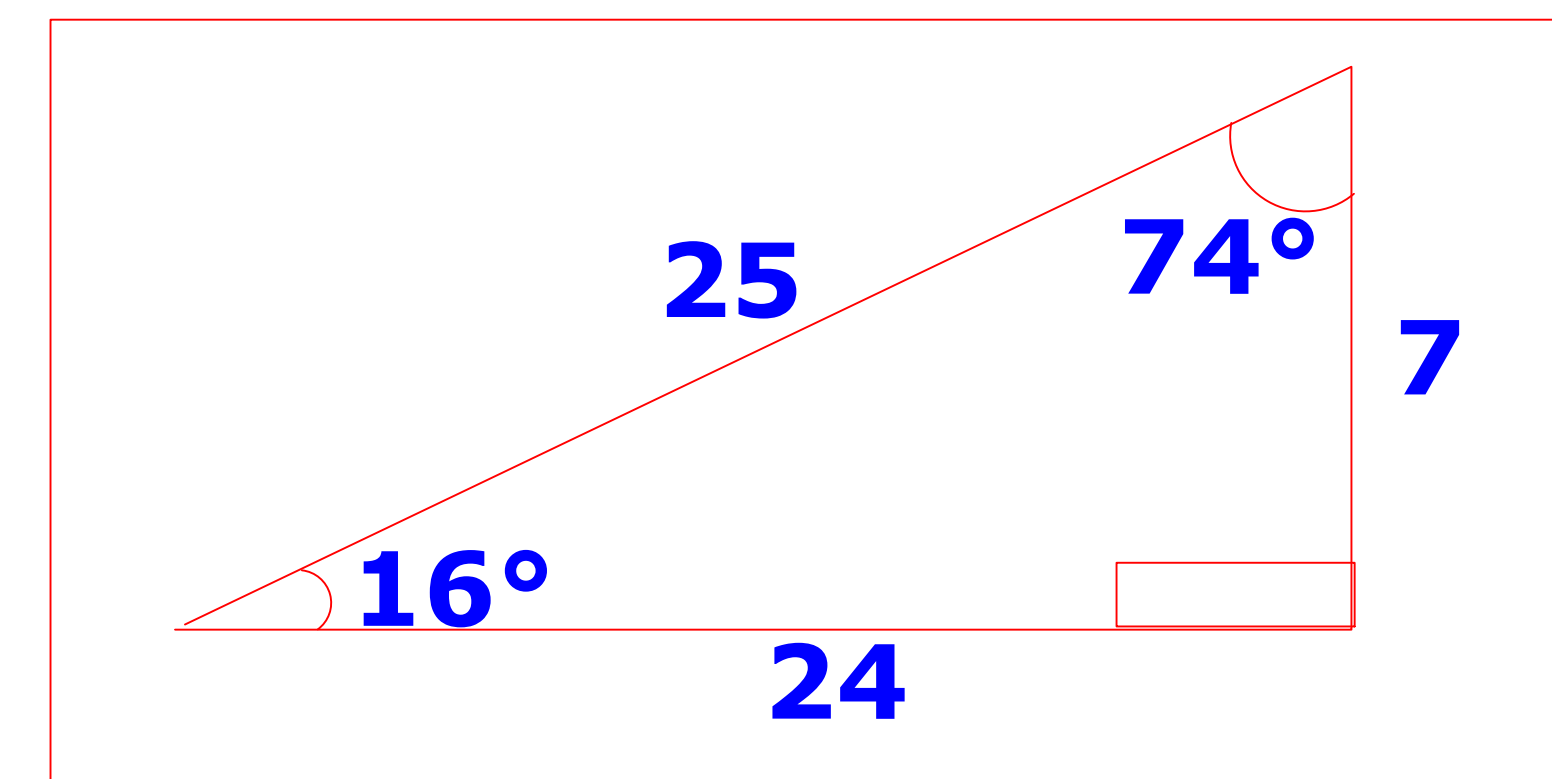
RESOLUCIÓN

$$\text{Sen}(x-y) = \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x$$

$$\text{Sen}(53^\circ - 37^\circ) = \text{Sen}53^\circ \cdot \text{Cos}37^\circ - \text{Sen}37^\circ \cdot \text{Cos}53^\circ$$

$$\begin{aligned}&= \frac{4}{5} \cdot \frac{4}{5} - \frac{3}{5} \cdot \frac{3}{5} \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \frac{16-9}{25} \\ &= \frac{7}{25}\end{aligned}$$

$$\text{Sen}16^\circ = \frac{7}{25}$$



$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

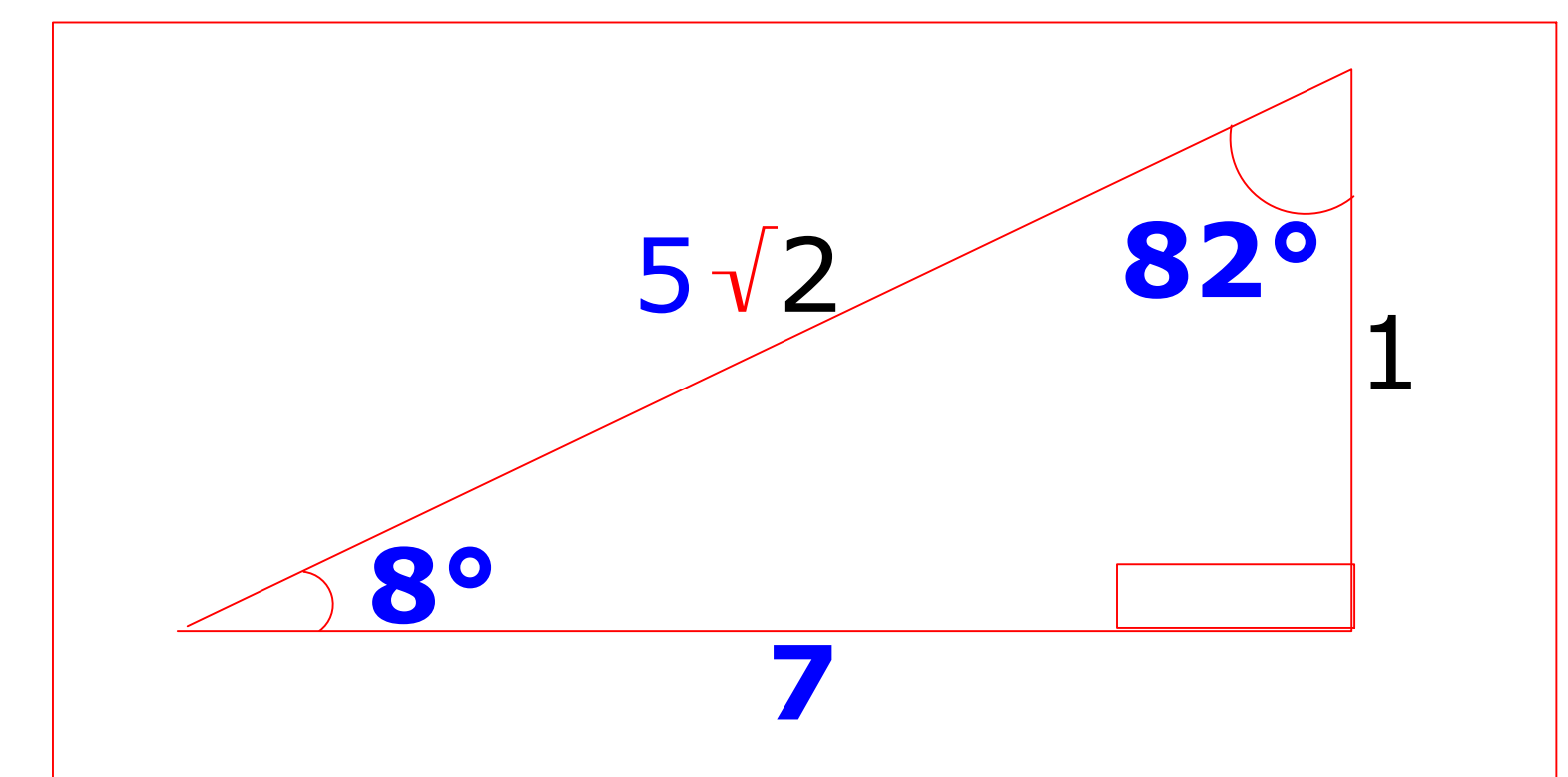
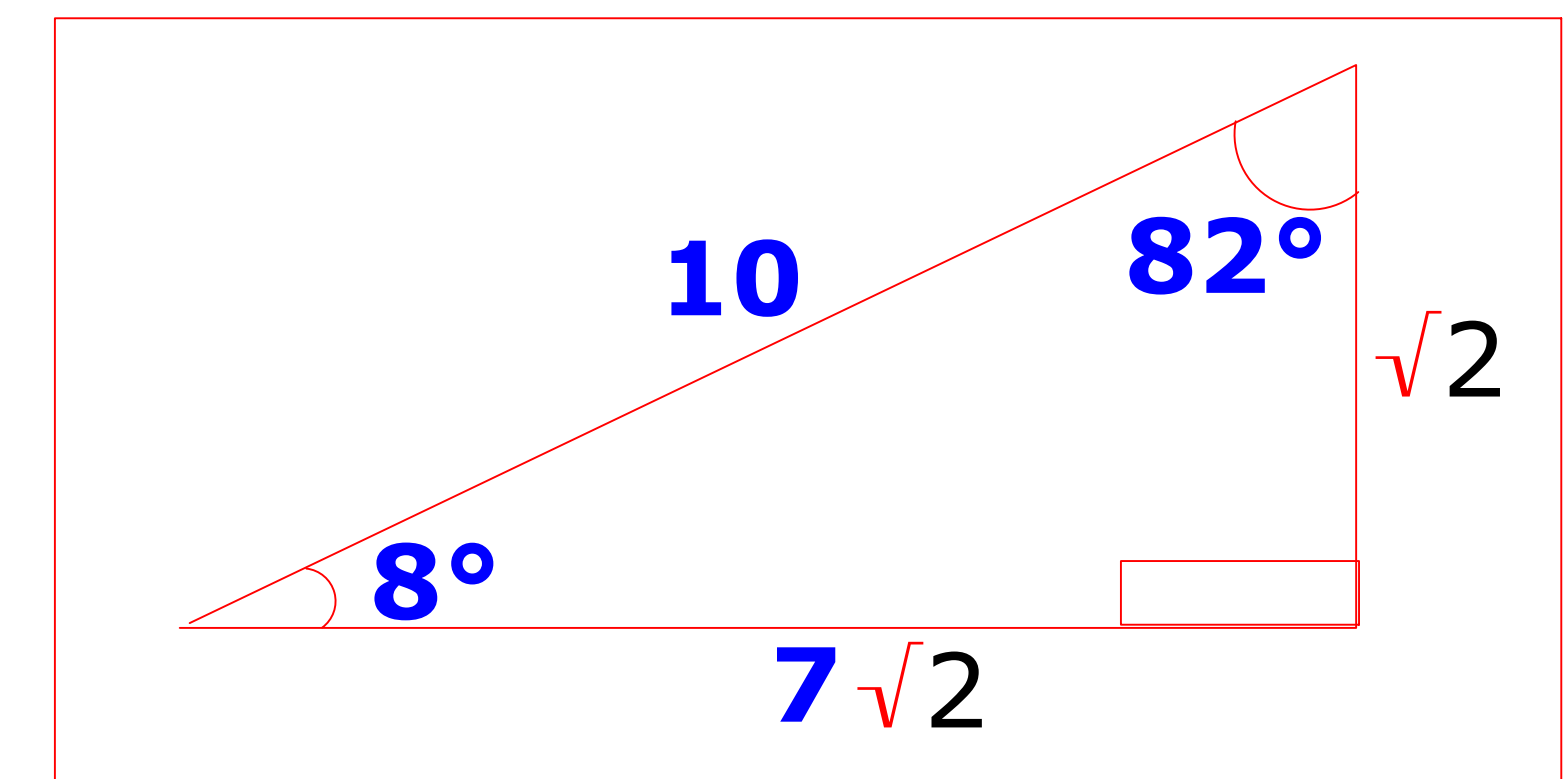
Calcular el valor de: $\text{Sen}(8^\circ)$

RESOLUCIÓN

$$\text{Sen}(x-y) = \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x$$

$$\begin{aligned}\text{Sen}(45^\circ - 37^\circ) &= \text{Sen}45^\circ \cdot \text{Cos}37^\circ - \text{Sen}37^\circ \cdot \text{Cos}45^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \left(\frac{4}{5}\right) - \left(\frac{3}{5}\right) \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{10}\end{aligned}$$

$$\text{Sen}8^\circ = \frac{\sqrt{2}}{10}$$



$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

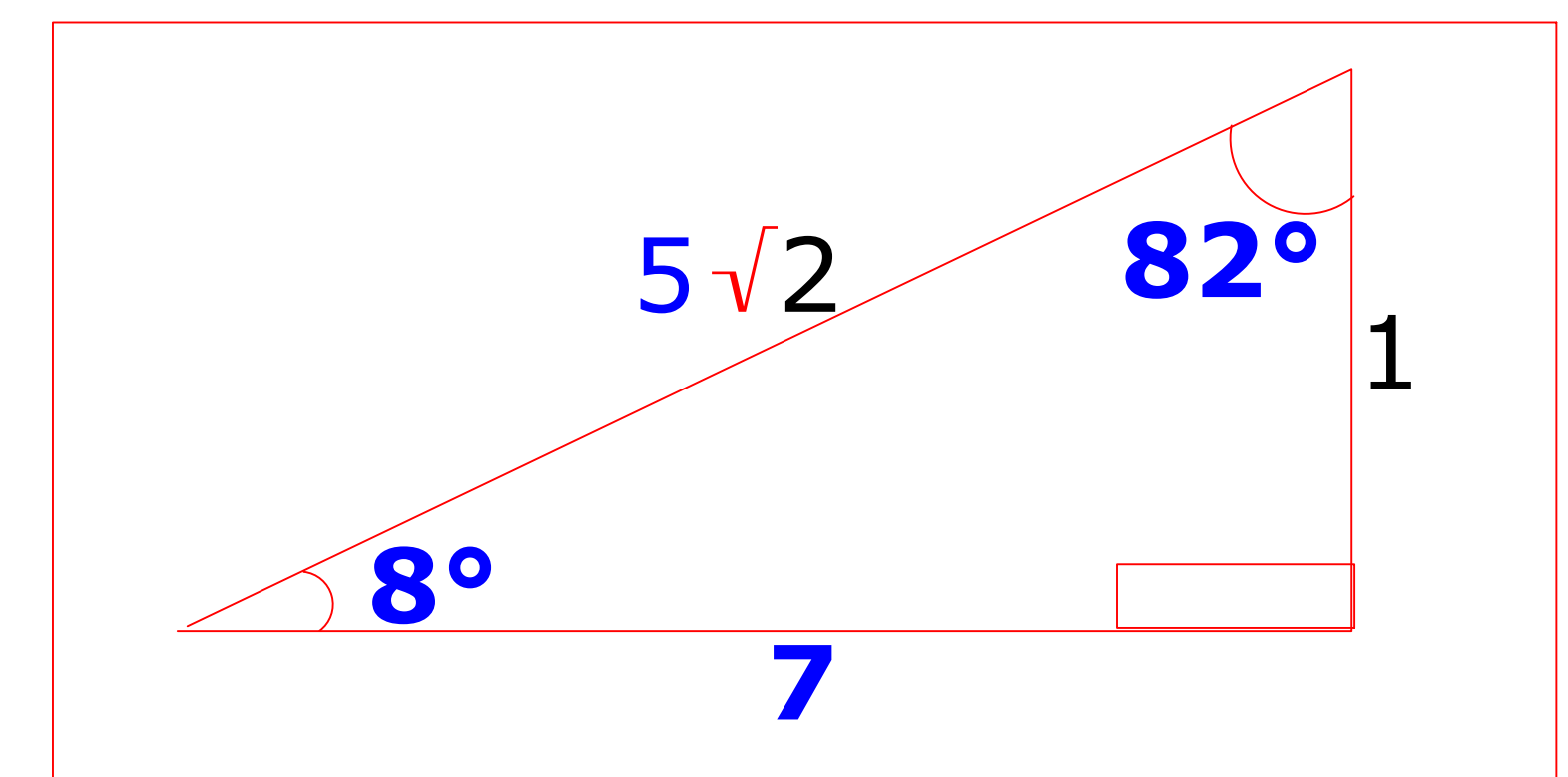
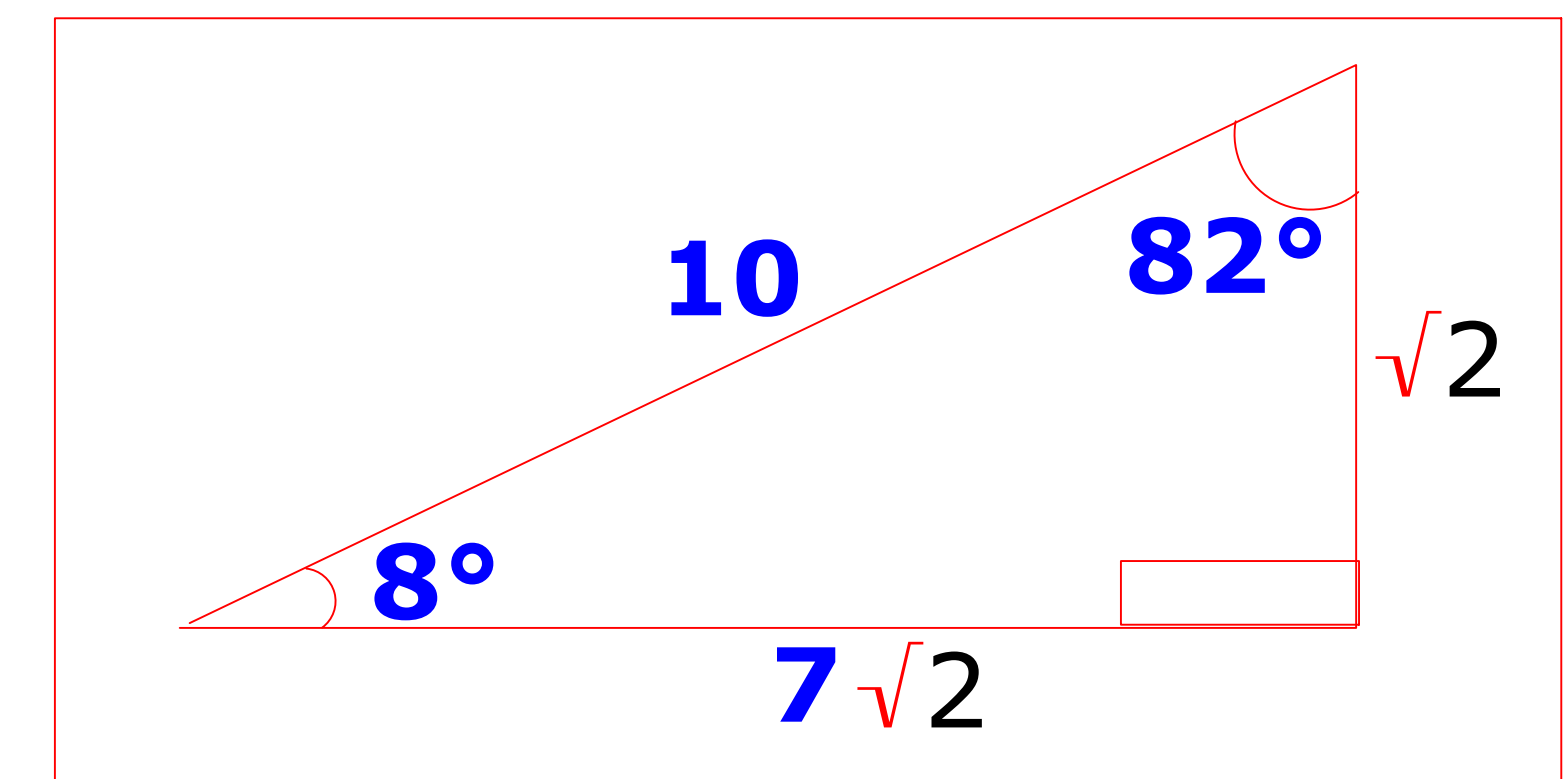
Calcular el valor de: $\text{Sen}(8^\circ)$

RESOLUCIÓN

$$\text{Sen}(x-y) = \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x$$

$$\begin{aligned}\text{Sen}(45^\circ - 37^\circ) &= \text{Sen}45^\circ \cdot \text{Cos}37^\circ - \text{Sen}37^\circ \cdot \text{Cos}45^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \left(\frac{4}{5}\right) - \left(\frac{3}{5}\right) \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{10}\end{aligned}$$

$$\text{Sen}8^\circ = \frac{\sqrt{2}}{10}$$



$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Halle el valor de:

$$E = \text{Sen}30^\circ \cdot \text{Cos}7^\circ + \text{Cos}30^\circ \cdot \text{Sen}7^\circ$$

RESOLUCIÓN

$$\text{Sen}(x+y) = \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x$$

$$E = \text{Sen}(37^\circ) = \frac{3}{5}$$

Halle el valor de x si:

$$\text{Cos}x \cdot \text{Cos}10^\circ - \text{Sen}x \cdot \text{Sen}10^\circ = \text{Cos}80^\circ$$

RESOLUCIÓN

$$\text{Cos}(x+y) = \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y$$

$$\cancel{\text{Cos}(x+10^\circ)} = \cancel{\text{Cos}80^\circ}$$

$$x+10^\circ = 80^\circ$$

$$x = 70^\circ$$

$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

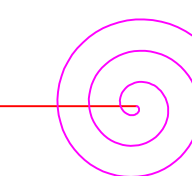
Determine el valor de:

$$P = \frac{\text{Sen}3x \cdot \text{Cos}4x + \text{Cos}3x \cdot \text{Sen}4x}{\text{Sen}5x \cdot \text{Cos}2x + \text{Cos}5x \cdot \text{Sen}2x}$$

RESOLUCIÓN

$$\text{Sen}(x+y) = \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x$$

$$P = \frac{\text{Sen}(3x+4x)}{\text{Sen}(5x+2x)} = \frac{\text{Sen}(7x)}{\text{Sen}(7x)} = 1$$



$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

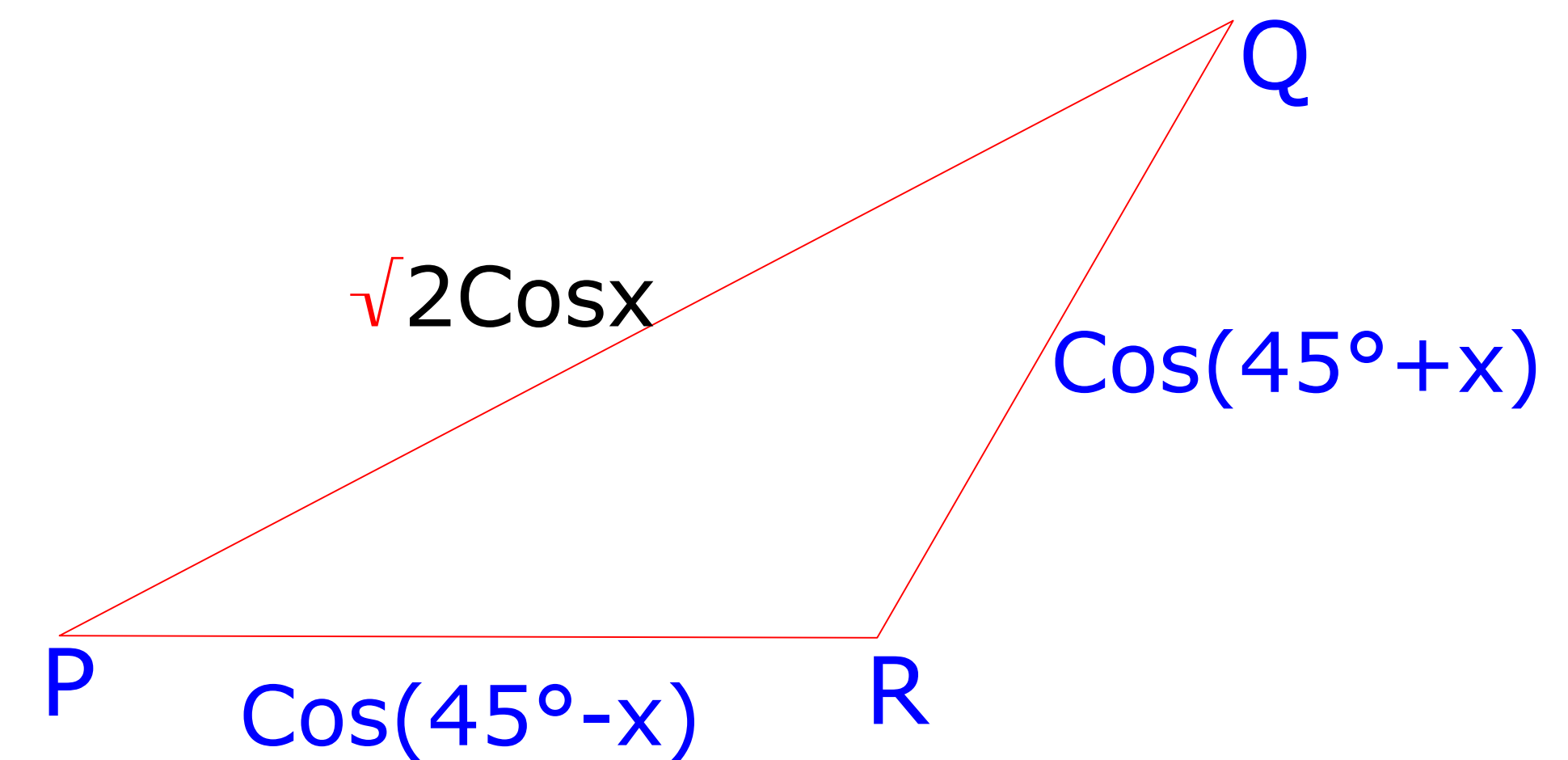
$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

$$\text{Cos}(x+y) + \text{Cos}(x-y) = 2 \text{Cos}x \cdot \text{Cos}y$$

Observe y determine el perímetro del triángulo PQR en términos de x.



RESOLUCIÓN

$$\text{Cos}(45^\circ+x) + \text{Cos}(45^\circ-x) = 2\text{Cos}45^\circ \cdot \text{Cos}x$$

$$2p = 2\text{Cos}45^\circ \cdot \text{Cos}x + \sqrt{2}\text{Cos}x$$

$$2p = 2 \cdot \frac{\sqrt{2}}{2} \cdot \text{Cos}x + \sqrt{2}\text{Cos}x = 2\sqrt{2}\text{Cos}x$$

$$2p = 2\sqrt{2}\text{Cos}x$$

$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Si: $\text{Tan}(x)=2$ y $\text{Tan}y = 3$.
Calcule: $\text{Tan}(x+y)$

RESOLUCIÓN

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\text{Tan}(x+y) = \frac{2 + 3}{1 - 2 \cdot 3}$$

$$\text{Tan}(x+y) = \frac{5}{1 - 6}$$

$$\text{Tan}(x+y) = \frac{5}{-5}$$

$$\text{Tan}(x+y) = -1$$

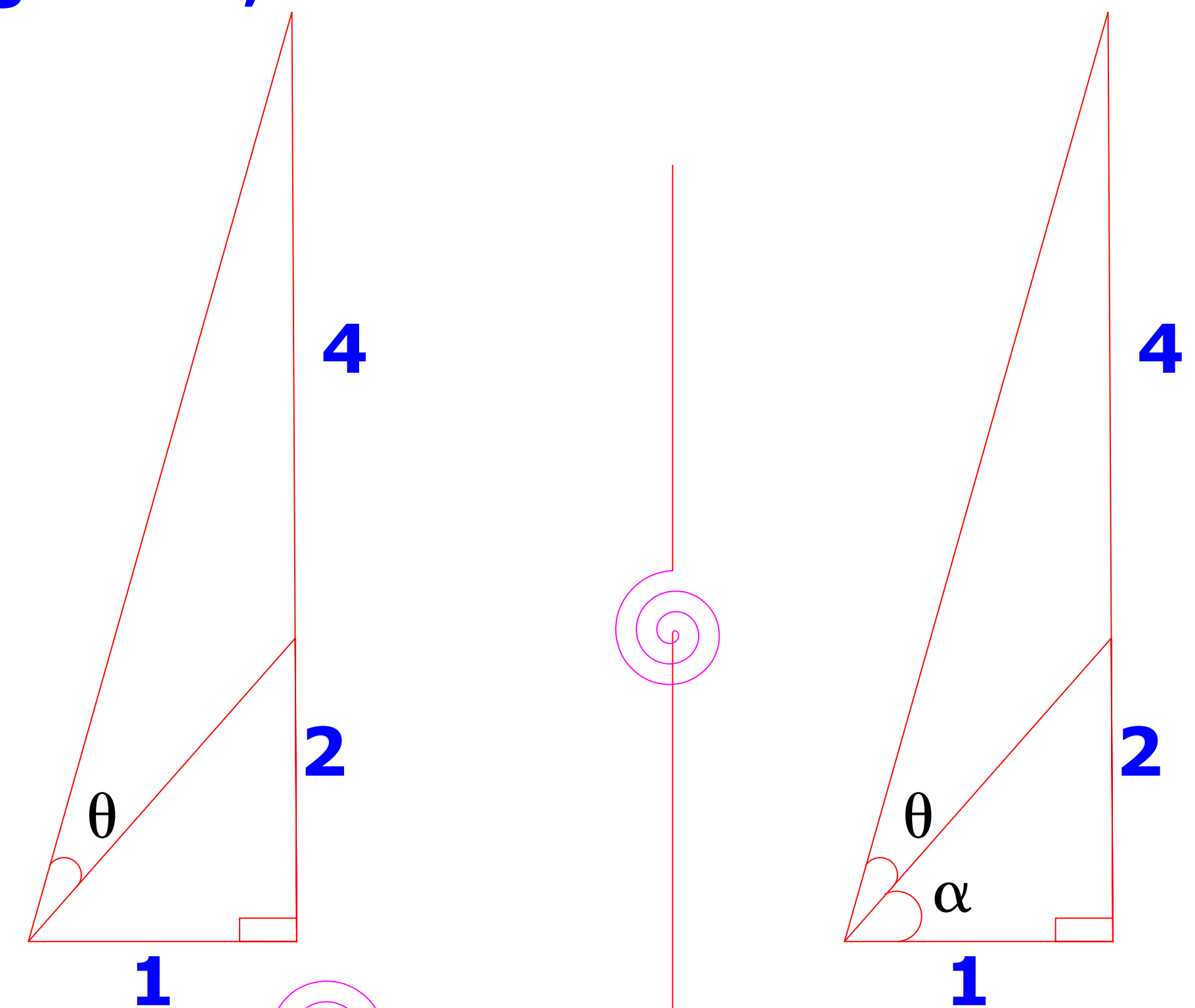
$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Del gráfico, calcule: $\text{Tan}\theta$



$$\text{Tan}\alpha = 2$$

$$\text{Tan}(\alpha + \theta) = \frac{\text{Tan}\alpha + \text{Tan}\theta}{1 - \text{Tan}\alpha \cdot \text{Tan}\theta}$$

Reemplazando
valores

$$6 = \frac{2 + x}{1 - 2x}$$

$$6(1-2x) = 2+x \Rightarrow 6 - 12x = 2+x$$

$$\begin{aligned}\Rightarrow 4 &= 13x \\ x &= \frac{4}{13}\end{aligned}$$

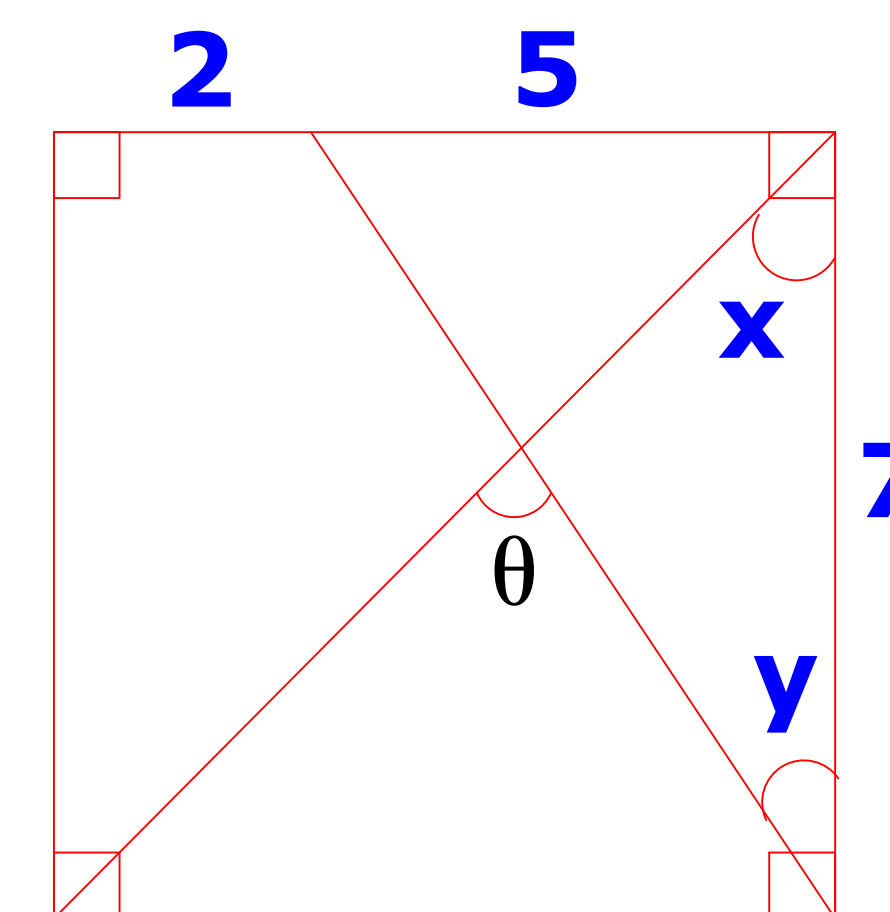
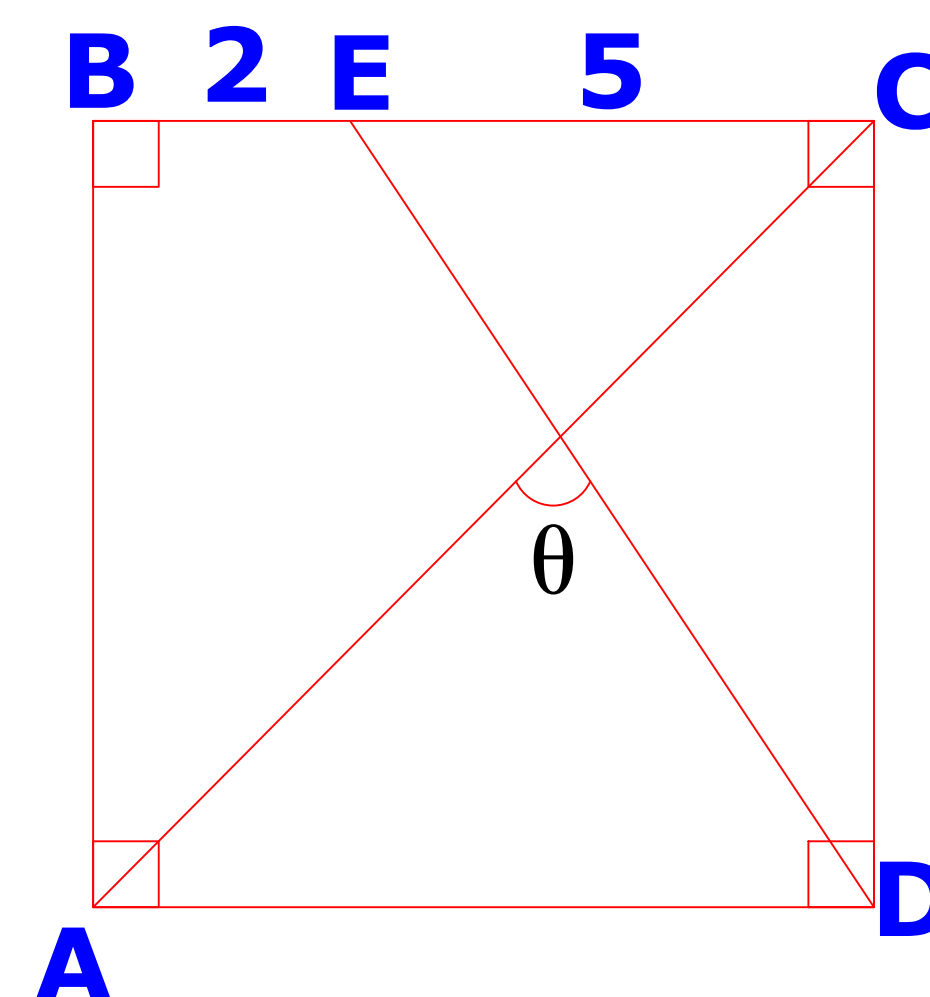
$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Del gráfico, calcule $\text{Tan}\theta$ si ABCD es un cuadrado



$$\theta = x + y$$

$$\text{Tan}x = 1; \text{Tan}y = \frac{5}{7}$$

$$\text{Tan}\theta = \text{Tan}(x + y)$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\text{Tan}(x+y) = \frac{1 + \frac{5}{7}}{1 - \frac{5}{7}}$$

$$\text{Tan}(x+y) = \frac{\frac{12}{7}}{\frac{2}{7}}$$

👍 Rta: $\text{Tan}\theta = 6$

$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tan}y}{1 - \text{Tan}x \cdot \text{Tan}y}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

Calcule Tanx, siendo $\tan(45^\circ + x) = 3$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tan}y}{1 + \text{Tan}x \cdot \text{Tan}y}$$

$$\frac{1 + \text{Tan}x}{1 - \text{Tan}x} = 3$$

$$1 + \text{tan}x = 3(1 - \text{Tan}x)$$

$$1 + \text{Tan}x = 3 - 3\text{Tan}x$$

$$4\text{Tan}x = 2$$

$$\text{Tan}x = \frac{1}{2}$$

 Rta: $\text{Tan}x = \frac{1}{2}$

$$\begin{aligned}\text{Sen}(x+y) &= \text{Sen}x \cdot \text{Cos}y + \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x+y) &= \text{Cos}x \cdot \text{Cos}y - \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tany}}{1 - \text{Tan}x \cdot \text{Tany}}$$

$$\begin{aligned}\text{Sen}(x-y) &= \text{Sen}x \cdot \text{Cos}y - \text{Sen}y \cdot \text{Cos}x \\ \text{Cos}(x-y) &= \text{Cos}x \cdot \text{Cos}y + \text{Sen}x \cdot \text{Sen}y\end{aligned}$$

$$\text{Tan}(x-y) = \frac{\text{Tan}x - \text{Tany}}{1 + \text{Tan}x \cdot \text{Tany}}$$

Si se cumple que $\text{Tan}(x+y) = \frac{3}{5}$ y $\text{Tan}x = \frac{1}{2}$,
Calcule tany.

$$\text{Tan}(x+y) = \frac{\text{Tan}x + \text{Tany}}{1 - \text{Tan}x \cdot \text{Tany}}$$

$$\frac{1/2 + \text{Tany}}{1 - 1/2 \cdot \text{Tany}} = \frac{3}{5}$$

$$5\left(\frac{1}{2} + \text{Tany}\right) = 3\left(1 - \frac{\text{Tany}}{2}\right)$$

$$\frac{5}{2} + 5\text{Tany} = 3 - \frac{3\text{Tany}}{2}$$

$$5\text{Tany} + \frac{3\text{Tany}}{2} = 3 - \frac{5}{2}$$

$$\frac{13\text{Tany}}{2} = \frac{1}{2}$$

$$13\text{Tany} = 1$$

$$\text{Tany} = \frac{1}{13}$$

👍 Rta: $\text{Tany} = \frac{1}{13}$